

The Björling problem for minimal surfaces in a Lorentzian 3-dimensional Lie group

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The Weierstrass representation formula for minimal surfaces in $\mathbb{R}^3$ has been a fundamental tool for producing examples and proving general properties of such surfaces, since the surfaces can be parametrized by holomorphic data. In [4] the authors describe a general Weierstrass representation formula for simply connected minimal surfaces in an arbitrary Riemannian manifold. The classical Björling problem, proposed by Björling in 1844, asks for the construction of a minimal surface in $\mathbb{R}^3$ containing a given analytic curve β with a given analytic unit normal V along it. The problem was solved by H.A. Schwarz in 1890 by means of an integral formula in terms of β and V . We show how the Weierstrass representation formula can be used, if the ambient manifold is a 3-dimensional Lorentzian Lie group, in order to prove existence and uniqueness of the solution of the Björling problem.

References

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