Self-similar solutions to the curvature flow and its inverse on the 2-dimensional light cone

Abstract

We consider a novel curvature flow for curves on the 2-dimensional light cone, contained in the 3-dimensional Minkowski space. We show that the solutions to the curvature flow (CF) for such curves are in correspondence with the solutions to the inverse curvature flow (ICF). We prove that the ellipses and the hyperbolas are the only curves that evolve under homotheties. The ellipses are the only closed ones and they are ancient solutions. We show that a spacelike curve on the cone is a self-similar solution to the CF (resp. (ICF)) if, only if, its curvature (resp. inverse of its curvature) differs by a constant c from being the inner product between its tangent vector field and a fixed vector v of the 3-dimensional Minkowski space. The curve is a soliton solution when $c = 0$. We prove that for each vector v there exists a 2-parameter family of self-similar solutions to the CF and to the ICF, on the light cone. Considering non-trivial solutions to the CF, we prove that the corresponding solutions to the ICF may have at most three connected components. Moreover, at each end of such a curve the curvature is either unbounded or it tends to 0 or to the constant c.

References

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