Curve Shortening Flow on $\mathbb{T}^2$

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Abstract

We present a characterization for the initial curve of a soliton solution for the curve shortening flow (CSF) on the torus of revolution $\mathbb{T}^2 \subset \mathbb{R}^3$. Furthermore, we describe the behavior of such curves by showing that the two ends of each curve are asymptotic to the equator.

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