

THE COMPLETE BIFURCATION DIAGRAM OF A KIRCHHOFF-TYPE EQUATION

KAYE SILVA *

Abstract. Consider the following Kirchhoff-type equation:

$$\begin{cases} -\left(a + \lambda \int |\nabla u|^2\right) \Delta u = |u|^{\gamma-2}u & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \quad (\text{K})$$

where $a > 0$, $\lambda > 0$ is a parameter, $\gamma \in (2, 4)$ and $\Omega \subset \mathbb{R}^3$ is a bounded regular domain. We give a description of the bifurcation diagram of (K) with respect to λ , only by using variational methods. In fact, we extend and complete the results of [1] by doing a finer analysis with respect to the Nehari set associated to (K) and proving the existence of a parameter $\lambda^* > 0$ such that: if $\lambda \in (0, \lambda^*)$ problem (K) has at least two non-zero solutions; if $\lambda = \lambda^*$ it has at least one non-zero solution and if $\lambda > \lambda^*$ there is no non-zero solution at all.

References

- [1] Kaye Silva, *The bifurcation diagram of an elliptic Kirchhoff-type equation with respect to the stiffness of the material*, Z. Angew. Math. Phys (To Appear).