Oscillatory regimes in resonant symmetric 3D piecewise-linear Filippov systems

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Resumo

This seminar presents a classification of the oscillatory regimes of simple period in a resonant symmetric class of three-dimensional piecewise-linear Filippov systems $Z = (X, Y)$. The two affine vector fields are related by an involution, and the switching plane contains a visible–visible two-fold singularity. In canonical coordinates, the complex-conjugate eigenvalues of one side are $C \pm i$, with $C > 0$, the real eigenvalue is $-2C$, and the parameter H measures the inclination of the corresponding focal line on the switching plane.

For every fixed $C > 0$, we determine an open interval $\mathcal{I}_C$ such that a crossing periodic orbit of simple period exists if and only if $H \in \mathcal{I}_C$. Every periodic orbit of simple period in this class is symmetric, and for each admissible value of H the corresponding orbit is unique within the simple-period class. Moreover, its half-period parametrizes $\mathcal{I}_C$ bijectively. Consequently, the system parameters determine the crossing points, the periodic orbit, its period, and its transverse spectral data.

The classification relies on two structural features of the system. Symmetry reduces the full return map to the square of a half-return map, while resonance provides polynomial first integrals whose matching reduces the closure problem to the geometry of a conic on the switching plane. This representation also allows the stability of the classified oscillations to be determined: every finite-amplitude orbit in the admissible regime is isolated, hyperbolic, and orbitally asymptotically stable. The endpoints of $\mathcal{I}_C$ correspond respectively to grazing and to a $+1$ spectral boundary attained only in the infinite-amplitude limit.

Finally, the hyperbolicity of each finite-amplitude cycle and the transversality of its intersections with the switching plane imply persistence under sufficiently small sidewise C^1 perturbations. A circuit realization will also be discussed, translating the classified oscillatory regime into explicit relations among circuit components.