ON THE DULAC'S PROBLEM FOR TANGENTIAL POLYCYCLES *

KAMILA ANDRADE †

Abstract

The well known Hilbert's 16^{th} problem gave rise to a lot of works and has motivated many researchers until nowadays. A first step towards the solution of this problem is to prove the following affirmation:

- *A polynomial vector field on* $\mathbb{R}^2$ *has at most a finite number of limit cycles.*

This question was first studied by Dulac in 1923 who gave an incomplete proof, which was noticed later. This finiteness question can be reduced to the problem of non-accumulation of limit cycles for a polynomial vector field, called *Dulac's problem*:

- *An elementary polycycle of an analytic vector field* Z *cannot have limit cycles accumulating onto it.*

In short, a polycycle is a closed oriented curve formed by a finite union of regular orbits and singular points of Z. A polycycle is said to be elementary if all its singular points are elementary singularities, basically hyperbolic saddles and saddle-nodes, see more in [4, 5]. Based on this, a question arise:

- *What happens if* Z *is a piecewise analytic vector field?*

Consider piecewise analytic vector fields in $\mathbb{R}^2$ of the form

$$Z(p) = \begin{cases} X(p), & p \in \Sigma^+, \\ Y(p), & p \in \Sigma^-, \end{cases}$$

where X and Y are analytic vector fields in $\mathbb{R}^2$,

$$\Sigma^+ = \{p \in \mathbb{R}^2; h(p) > 0\} \qquad \text{and} \qquad \Sigma^- = \{p \in \mathbb{R}^2; h(p) < 0\}$$

and $h : \mathbb{R}^2 \to \mathbb{R}$ is a smooth function for which 0 is a regular value. The set $\Sigma = \{p \in \mathbb{R}^2; h(p) = 0\}$ is called switching curve and a vector field is denoted by $Z = (X, Y)$, see more in [2, 3]. In this context, a polycycle of a piecewise analytic vector field $Z = (X, Y)$ is also a closed oriented curve composed by regular trajectories and singularities. The main and crucial difference between this and the analytic case lies in the fact that new types of singularities can be considered, for example one can consider points in Σ which are regular for X and singular for Y, or singular for both X and Y but of different type. Here, a singular point is an usual equilibrium point or a point where one of the vector fields X and Y is tangent to Σ, called tangential singularity. In, [1] authors prove that a polycycle which singularities are only hyperbolic saddles, second order tangent points, or/and combined of this two types cannot have limit cycles accumulating onto it.

The main objective of this seminar is to prove that tangential polycycles, i.e., polycycles composed only by tangential singularities and regular trajectories cannot have limit cycles accumulating on it.

References

[1] K. DA S. ANDRADE, M. R. JEFFREY, R. M. MARTINS, M. A. TEIXEIRA - On the Dulac's problem for piecewise analytic vector fields. *Journal of Differential Equations* **266** (2019), 2259-2273.

*Joint work with O. M. L. Gomide

†Instituto de Matemática e Estatística, UFG, kamila.andrade@ufg.br

[2] A.F. Filippov - Differential Equations with Discontinuous Right-Hand Sides. *Math. Appl. (Sov. Ser.)*, vol **18**, Kluwer Academic Publishers Group, Dordrecht, 1988.

[3] M. Guardia, T.M. Seara, M. A. Teixeira - Generic Bifurcations of low codimension of Planar Filippov Systems. *J. Differential Equations* **250** (2011), 1967-2023.

[4] Yu. Il'Yashenko - Limit cycles of polynomial vector fields with nondegenerate singular points on the real plane. *Anal. Ego. Pri.*, **18**, 3, (1984), 32-34. (Func. Ana. and Appl., 18, 3 (1985), 199-209).

[5] R. Roussarie - Bifurcation of Planar Vector Fields and Hilbert's Sixteenth Problem. *Birkhäuser*, 1998.