

Periodic orbits in slow–fast piecewise continuous linear Hamiltonian systems

Abstract

We prove analytically the existence of crossing periodic orbits for a family of piecewise continuous linear slow-fast Hamiltonian systems defined in $\mathbb{R}^4$ when a parameter $\varepsilon > 0$ is sufficiently small. This system is derived from a differential equation that serves as a model for describing phenomena in physics. We provide a generic necessary condition for the existence of crossing one–turn periodic orbits. We discuss the relationship between the crossing one–turn periodic orbits and the one–turn homoclinic orbit existing in this system. In addition, for $\varepsilon > 0$ sufficiently small we investigate the existence and non-existence of crossing one–turn periodic orbits through appropriately chosen subsets of the switching manifold. This work presents original ideas and an analytical method based on the concept of uniform convergence. The system we study is the following:

$$\begin{aligned}\varepsilon \dot{x} &= y, \\ \varepsilon \dot{y} &= -x - w, \\ \dot{z} &= x, \\ \dot{w} &= \begin{cases} -z/a, & \text{if } z \leq a, \\ (z-1)/(1-a), & \text{if } z \geq a; \end{cases}\end{aligned}$$

where $0 < a < 1$ and $\varepsilon > 0$ small.