Does an isolated quantum many-body system relax?

Workshop on Quantum Information and Thermodynamics – 2015-02-25

Bernhard Rauer

Schmiedmayer Group, Atominstitut, TU Vienna
Vienna Center for Quantum Science and Technology
Start from an isolated quantum system in thermal equilibrium
Motivation

Non-equilibrium state

Quench: $H \rightarrow H'$

$\psi_0 \rightarrow \psi_0$

Thermal equilibrium
Motivation

Non-equilibrium state

Non-equilibrium dynamics of closed quantum system

$\psi_0 \rightarrow \psi(t)$

Recurrent unitary evolution?
Relaxation on a single timescale?
Multiple timescales?
Thermalization?

Thermal equilibrium (total loss of memory)
Experimental system: 1D Bose gas

1D Bose gas on an atom chip:

- Precise control over system parameters
- Near perfect isolation from the environment
- Direct probes through absorption imaging

Experimental parameters:

- Weakly interacting regime
- 2000 – 10 000 atoms of Rb\textsuperscript{87}
- Temperature of 20 – 100 nK
- Trap frequencies

\[
\begin{align*}
\omega_{\perp} &= 2\pi \cdot 2 \ k Hz \\
\omega_{||} &= 2\pi \cdot 10 \ Hz
\end{align*}
\]

\[\mu, \ k_B T < \hbar \omega_{\perp}\]
Start with a single, phase fluctuating 1D quasi-condensate
Experimental scheme

Start with a single, phase fluctuating 1D quasi-condensate

split it via RF dressed state potentials
Experimental scheme

Relative phase

\[ \Delta \phi(z) = \phi_1(z) - \phi_2(z) \]

Quench by coherent splitting
Experimental scheme

common modes:
high energy (thermal noise)

relative modes:
low energy (quantum noise)

\[ \phi(z) \]

\[ \Delta \phi(z) = \phi_1(z) - \phi_2(z) \]

\[ \lambda_T \]

\[ t = 0 \text{ ms} \]
Experimental scheme

$t = 0$ ms

$\lambda_T$

$\phi(z)$

$\Delta \phi(z) = \phi_1(z) - \phi_2(z)$

$t > 0$ ms

How does this state evolve in time?
Experimental scheme

Probe the system using matter-wave interference:

$$\phi(z)$$

relative phase

$$\Delta \phi(z) = \phi_1(z) - \phi_2(z)$$
Experimental scheme

Probe the system using matter-wave interference: direct access to the relative phase field!

\[ \Delta \phi(z) = \phi_1(z) - \phi_2(z) \]
Questions to answer

- Does the system relax?
- How does this relaxation happen?
- Can we describe the relaxed state in terms of statistical ensembles?
- Does the system eventually thermalize?
- How can evaporative cooling work in these systems?
Questions to answer

- Does the system relax?
- How does this relaxation happen?
- Can we describe the relaxed state in terms of statistical ensembles?
- Does the system eventually thermalize?
- How can evaporative cooling work in these systems?
Does the system relax?

initial state

$\lambda \to \infty$

$\Delta \phi(z)$

mean squared contrast $<C^2>$

time [ms]
Does the system relax?

Rapid emergence of a steady state with thermal correlations!

Extracted temperature an order of magnitude smaller than initial one.

e.g.: \( T_{\text{eff}} = 11\text{nK} \)
\( T_{\text{in}} = 100\text{nK} \)
Does the system relax?

Rapid emergence of a steady state with thermal correlations!

Relaxation through dephasing of the initial quantum fluctuations → Prethermalization

Initial state

\[ \lambda \to \infty \]
\[ \Delta \phi(z) \]

Relaxed state

\[ \lambda = \lambda_{T_{eff}} \]

Extracted temperature an order of magnitude smaller than initial one.

e.g.:
\[ T_{eff} = 11\text{nK} \]
\[ T_{in} = 100\text{nK} \]

two in thermal equilibrium, where \[ T_{rel} = T_{com} \] shows: relaxed state is not thermal equilibrium

Science, 337, 1318 (2012)
Questions to answer

- Does the system relax?
- How does this relaxation happen?
- Can we describe the relaxed state in terms of statistical ensembles?
- Does the system eventually thermalize?
- How can evaporative cooling work in these systems?
Relaxation dynamics

initial state

\[ \lambda \to \infty \]

\[ \Delta \phi(z) \]

prethermalized state

\[ \lambda = \lambda_{T_{eff}} \]

How are the initial correlations lost?

What do we see if we look at the local dynamics?
Relaxation dynamics

initial state

$\lambda \to \infty$

$\Delta \phi(z)$

prethermalized state

$\lambda = \lambda_{eff}$

$C(z, z') = \langle e^{i \phi(z) - i \phi(z')} \rangle$

correlation function, $C(\bar{z})$

relative distance, $\bar{Z}$ (µm)
Relaxation dynamics

Initial state

\[ \lambda \rightarrow \infty \]

\[ \Delta \phi(z) \]

Prethermalized state

\[ \lambda = \lambda_{Eff} \]

Graph showing correlation function, \( C(\bar{Z}) \) vs. relative distance, \( \bar{Z} \) (μm).
Relaxation dynamics

![Graph showing correlation function C(\bar{z}) against relative distance \bar{z} = z - z' (\mu m) with a time label of 1 ms.](image)
Relaxation dynamics

![Graph showing correlation function $C(\tilde{z})$ over relative distance $\tilde{z} = z - z'$ (µm) with points at 1 ms and 2 ms.]
Relaxation dynamics
Relaxation dynamics

![Graph showing relaxation dynamics over time]

- Time points: 1 ms, 2 ms, 3 ms, 4 ms
- Correlation function $C(\tilde{z})$
- Relative distance $\tilde{z} = z - z'$ (µm)
Relaxation dynamics

![Graph showing correlation function C(\tilde{z}) vs. relative distance \tilde{z} = z - z' (\mu m) with time markers at 1 ms, 2 ms, 3 ms, 4 ms, 5 ms, and 10 ms.]
Relaxation dynamics

Beyond $z_c$, long-range order remains.

Thermal decay up to a characteristic distance $z_c$.

Prethermalized state.
Local relaxation dynamics

Thermal correlations emerge locally and spread in a light-cone like evolution!
Local relaxation dynamics

Thermal correlations emerge locally and spread in a light-cone like evolution!

characteristic velocity of the correlation front:

\[ z_c = 2c t \]

quasi-particles pairs propagate information in opposite directions

Questions to answer

- Does the system relax?
- How does this relaxation happen?
- Can we describe the relaxed state in terms of statistical ensembles?
- Does the system eventually thermalize?
- How can evaporative cooling work in these systems?
1D Bose gas with contact interactions is an integrable system with many conserved quantities.

\[ \hat{\rho} = \frac{1}{Z} \exp \left( - \sum_m \lambda_m I_m \right) \]

Lagrange multipliers:
\[ \lambda_m = \beta_m = \frac{1}{k_B T_m} \]

generalized temperatures

Integrable systems are conjectured to relax to a maximum entropy state described by a generalized Gibbs ensemble (GGE):

→ inhibited thermalization

Many parameters needed to describe the thermal state!
Full 2-point correlations

Non-translation-invariant phase correlation function:
Full 2-point correlations

Non-translation-invariant phase correlation function:

Light-cone dynamics:
Full 2-point correlations

Non-translation-invariant phase correlation function:

Light-cone dynamics:

Previous correlation functions were cuts through this full two-point function
Full 2-point correlations

Non-translation-invariant phase correlation function:

Light-cone dynamics:

Dynamics and steady state can be described by one temperature!
Observation of a GGE

Different initial state by modified splitting process:

clearly visible cross-structure in the correlation function
Observation of a GGE

Different initial state by modified splitting process:

- Clearly visible cross-structure in the correlation function

Stronger correlations for points equally far away from the center:

\[ \Delta \phi(z) \quad z_1 = -z_2 \]
Observation of a GGE

Different initial state by modified splitting process:

- Clearly visible cross-structure in the correlation function.

Stronger correlations for points equally far away from the center.

\[ \phi(z) \quad z_1 = -z_2 \]

Can be explained by an imbalanced population of even and odd modes.

At least 2 temperatures needed to describe this.

Direct observation of a GGE!

\[ \hat{\rho} = \frac{1}{Z} e^{-\sum_m \lambda_m I_m} \]
Observation of a GGE

Different initial state by modified splitting process:

\[ \chi^2 \text{ Analysis shows that the steady state is not compatible with a single temperature} \]

\[ \hat{\rho} = \frac{1}{Z} e^{-\sum_m \lambda_m I_m} \]
Observation of a GGE

Different initial state by modified splitting process:

\[ \hat{\rho} = \frac{1}{Z} e^{-\sum_m \lambda_m \mathcal{I}_m} \]

\( \chi^2 \) Analysis shows that the steady state is not compatible with a single temperature

Fitting the steady state reveals the individual mode occupations

With 10 fitted temperatures we can describe the steady state and the dynamics!

(number corresponds to what we expect from resolution and decreasing contribution of higher modes)

Langen et al. arXiv:1411.7185
Questions to answer

- Does the system relax?
- How does this relaxation happen?
- Can we describe the relaxed state in terms of statistical ensembles?
- Does the system eventually thermalize?
- How can evaporative cooling work in these systems?
Long-term dynamics

We clearly see a second much slower decay and the emergence of a further steady state!

Contrast evolution:

We think this can be attributed to a non-linear relaxation of the phonon modes.

Integrability does not have to be broken for this!

Stimming et al. PRA 83, 023618 (2011)
Proper quantitative analysis of the long-term evolution is challenging due to several reasons:

- Imbalanced splitting couples relative and common degrees of freedom
- Recurrent behavior of the system
- Excitations from the splitting process
- Atom loss

Furthermore, the trapped system is only nearly integrable! How does this integrability braking influence the dynamics? (extension of the KAM theorem to quantum mechanics?)
Questions to answer

- Does the system relax?
- How does this relaxation happen?
- Can we describe the relaxed state in terms of statistical ensembles?
- Does the system eventually thermalize?
- How can evaporative cooling work in these systems?
Motivation

Conventional evaporative cooling:

key ingredients:

- energy selective out-coupling of particles
- consecutive thermalization
Motivation

Conventional evaporative cooling:

- **energy selective out-coupling** of particles
- **consecutive thermalization**

1D Bose gas:

Energy selective removal of particles is in principle possible but **thermalization is inhibited**

→ should render cooling ineffective
Motivation

Conventional evaporative cooling:

key ingredients:
- energy selective out-coupling of particles
- consecutive thermalization

1D Bose gas:

Energy selective removal of particles is in principle possible but thermalization is inhibited → should render cooling ineffective

Nevertheless cooling is observed in experiment. Why?
Proposed mechanism

Homogeneous out-coupling of atoms scales down not only the average density but also the density fluctuations:

\[
\langle |n_k|^2 \rangle \rightarrow \Gamma^2 \langle |n_k|^2 \rangle \\
\langle |\phi_k|^2 \rangle \rightarrow \langle |\phi_k|^2 \rangle
\]

takes out energy from density quadrature creating a non-equilibrium state that dephases.
Homogeneous out-coupling of atoms scales down not only the average density but also the density fluctuations:

\[
\langle |n_k|^2 \rangle \rightarrow \Gamma^2 \langle |n_k|^2 \rangle \\
\langle |\phi_k|^2 \rangle \rightarrow \langle |\phi_k|^2 \rangle
\]

takes out energy from density quadrature creating a non-equilibrium state that dephases

In the limit of slow evaporation this results in a linear scaling of temperature and atom number

\[
T(t) = \left( \frac{n_0(t)}{n_0(0)} \right)^{3/2} T(0) = \frac{N(t)}{N(0)} T(0)
\]
Proposed mechanism

This simple model does not include the fluctuations introduced through the dissipation!

Treating the dissipation as a coupling to an empty bath leads to an effective stochastic Gross-Pitaevskii equation (SGPE):

\[
\left( i\partial_t + \frac{\partial^2}{2m} + \frac{g}{2} |\Phi|^2 + i\gamma \right) \Phi + \xi(t) = 0
\]
Proposed mechanism

This simple model does not include the fluctuations introduced through the dissipation!

Treating the dissipation as a coupling to an empty bath leads to an effective stochastic Gross-Pitaevskii equation (SGPE):

\[
\left( i\partial_t + \frac{\partial^2}{2m} + \frac{g}{2} |\Phi|^2 + i\gamma \right) \Phi + \xi(t) = 0
\]

homogeneous dissipation fluctuations

white noise correlations:

\[ \langle \xi^*(t)\xi(t') \rangle = 2\gamma \delta(t-t') \]
Proposed mechanism

This simple model does not include the fluctuations introduced through the dissipation!

Treating the dissipation as a coupling to an empty bath leads to an effective stochastic Gross-Pitaevskii equation (SGPE):

\[
\left( i \partial_t + \frac{\partial^2}{2m} + \frac{g}{2} |\Phi|^2 + i \gamma \right) \Phi + \xi(t) = 0
\]

homogeneous dissipation \hspace{2cm} fluctuations

\[
\langle \xi^*(t) \xi(t') \rangle = 2\gamma \delta(t - t')
\]

In the phononic limit this results in an additional term:

\[
k_B T(t) = \left( \frac{n_0(t)}{n_0(0)} \right)^{3/2} k_B T(0) + \mu(0) \left( \frac{n_0(t)}{n_0(0)} - \left( \frac{n_0(t)}{n_0(0)} \right)^{3/2} \right)
\]

classical \hspace{2cm} quantum fluctuations

Grisins et al. arXiv:1411.4946
Proposed mechanism

System runs into a dissipative state with temperature locked to the chemical potential.

\[
\frac{T}{\mu} \propto \frac{1}{\lambda T}
\]

ratio is a measure for coherence in the system.
Temperature measurement

Temperature is measured from the correlations of the longitudinal **density speckle pattern** forming in time-of-flight.

The gradient of the phase acts like the velocity local velocity:

\[ \vec{v} = \vec{\nabla}\phi(z) \]

Therefore the speckles are a direct result of the fluctuating phase in the trap.

From a fit with correlation functions created with simulated data we can **extract the thermal coherence length** \( \lambda_T \).
Measure temperature evolution under continuous dissipation of atoms: (Atoms are evaporated by RF transitions to untrapped states)

Temperature scaling with $N$

Measurements are consistent with the absence of quantum noise

possible explanation:
- homogeneous outcoupling too simple
- non-Markovianity in the outcoupling process
Conclusion and Summary

- Does the system relax?
  - dephasing to a prethermalized state

- How does this relaxation happen?
  - local emergence of thermal correlations

- Can we describe the relaxed state in terms of statistical ensembles?
  - observation of a generalized Gibbs ensemble

- Does the system eventually thermalize?
  - slow second decay that is hard to analyze

- How can evaporative cooling work in these systems?
  - down-scaling of density fluctuations through dissipation
Thank you!

Bernhard Rauer
Tim Langen
Thomas Schweigler
Maximilian Kuhnert
Michael Gring
Remi Geiger
David Adu Smith
Jörg Schmeidmayer

Theory:
Igor Mazets
Pjotrs Grisins
Sebastian Erne
Thomas Gasenzer
Takuya Kitagawa
Eugene Demler

Science 337, 1318 (2012)
Nature Phys. 9, 640 (2013)
PRL 110, 090405 (2013)
PRL 113, 190401 (2014)
NJP 15, 075011 (2013)
NJP 16, 053034 (2014)
EPJ, 217, 43 (2013)
arXiv:1411.4946
arXiv:1411.7185

FWF
Complex Quantum Systems
agute
Wien Kultur
EUROPEAN SCIENCE FOUNDATION
Siemens AG Österreich